Mission Function Control for Deployment and Retrieval of a Subsatellite

Hironori Fujii and Shintaro Ishijima
Tokyo Metropolitan Institute of Technology, Tokyo, Japan

This paper presents a new control algorithm applied to the problem of deployment and retrieval of a subsatellite connected through a tether to the main body. From control theory an idea of the "mission function" is introduced, which is an index used to describe the concept of the mission. The mission function is defined to be positive-definite, and to be zero when the given mission is completed. The deployment and retrieval is thus controlled to decrease the mission function. The control law is totally different from the control laws that have been presented in the literature; linearity and simple open-loop control are not assumed. In addition, the present theory can be applied equally to both the deployment and the retrieval cases. A simplified model is used to clarify the fundamentals of the problem, only in-plane motion is treated, and neither flexibility nor air drag is included. Results of numerical simulation show an excellent controlled behavior.

Introduction

DEPLOYMENT and retrieval is an essential phase for large space structures. However, this is usually a very complicated problem because the system is nonlinear and time varying. The structures are usually condensed because of the limited volume available in launchers, and so must be extended, deployed, constructed, or sometimes retrieved. Failure of this phase may impair the usefulness of the structure.

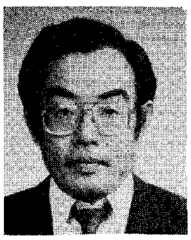
The example treated in this paper is deployment and retrieval of a subsatellite connected through a tether to the Shuttle.[1] Many important uses of tethered satellites have been identified.[2]

The dynamics and control of the system have been studied by many researchers and the literature on the control problem has been reviewed in a comprehensive work by Misra and Modi.[3] This review contains tables comparing dynamical models and comparing control laws for Shuttle-supported tethered systems used by various investigators until their review work in 1982; not many are left for citing in this paper. The dynamical models include a fundamentally simple model to a more realistic one. However, as Misra and Modi conclude, "Many questions regarding the control of the system, specifically during retrieval, remain unanswered."[3] The previously presented control laws do not seem to close their control loops. Their final states are not asymptotically stable in their control when the deployment or retrieval is accomplished and are critically affected by the initial conditions (for further detail, see Ref. 3).

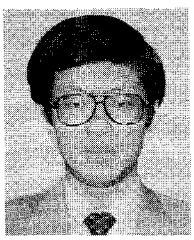
The retrieval of the tethered satellite is inherently unstable when one gives attention to the tether rotation, and its control is the most difficult phase during operation of the tethered satellite. Thus, Banerjee and Kane[4] and Xu et al.[5] propose to apply the thruster-augmented control to the retrieval of the tethered subsatellite because thrust augmentation and the use of transverse thrust help stabilize and speed up this process.

Because of the complexity of the problem, this paper employs a simplified model of the system to emphasize the fundamental aspects of the method. Thus, the flexibility and mass of the tether are neglected as are air drag and other atmospheric effects; only in-plane motion is treated.

During deployment and retrieval of the subsatellite, the tension force of the tether is controlled in order to change the tether length and to suppress oscillation of the tether. The control algorithm uses a mission function concept defining the mission objective. The mission function is positive-definite and is zero when the mission objective is completed. The deployment and retrieval are controlled to reduce the value of



Hironori Fujii is a professor in the Department of Aerospace Engineering at the Tokyo Metropolitan Institute of Technology. He earned his Ph.D. in engineering in 1975 from Kyoto University. In 1972, he became an associate professor, and in 1985 he was promoted to professor and became chairman of the Department of Aeronautical Engineering at the Metropolitan College of Technology. The college changed its system and became the present Institute in 1986. One of his activities since 1982 is coordinating the Research Group on Control of Flexible Space Structures in Japan. He is a Member of AIAA and JSASS, and an Associate Fellow of CASI.



Shintaro Ishijima was born in Saitama prefecture, Japan, in October 1947. He received B.E., M.E., and D.E. degrees from Waseda University in 1971, 1973, and 1976, respectively. He is currently a professor in the Department of Electronic Systems Engineering at the Tokyo Metropolitan Institute of Technology. His main research interests include analysis of nonlinear control systems, control of flexible structures, and development of CAD systems for controller design.

Received June 3, 1987; revision received Oct. 16, 1987.

the mission function, i.e., the time derivative of the mission function is forced to be negative-definite through the control process.

The control law obtained is naturally closed loop, and it is also nonlinear, which distinguishes it from others in the literature[3] that are open loop and sometimes linear. The present control is always stable. When this control algorithm is used, there is no basic difference between deployment and retrieval.

Description of the System

The system of a subsatellite connected to a main body (the Space Shuttle) is illustrated in Fig. 1. The center of attraction is denoted by point P and the center of mass of the main body by C. The orthogonal axes X and Y are defined along PC and along the orbital velocity vector, respectively, both originating at C. The parameters m, ℓ, and ϕ denote mass of the subsatellite, length of the tether, and position angle of the subsatellite in the orbital plane, respectively, as shown in Fig. 1.

Assumptions

Throughout the analysis the following assumptions are made:

1) The gravitational body P, the main body, and the subsatellite are particles.

2) The mass of the subsatellite is sufficiently small with respect to the main body that C always remains in its nominal orbit.

3) The attitude of the main body is controlled perfectly in the sense that motion of the subsatellite does not affect the attitude motion of the main body.

4) The tether has no mass and thus its flexibility is ignored.

5) The tether is assumed to be long compared to the length of the main body and the boom.

6) The only external force affecting the motion is the gravitational force caused by P. The orbit is circular and only motion in the orbital plane is considered.

7) The control force acts only along the tether through tension T and no control force or energy dissipation exists for motion perpendicular to the tether line.

The tethered subsatellite is deployed or retrieved through the tension control, i.e., the required tension is produced by the torque of a motor driving the tether reel system.[3]

Equations of Motion

Under the above assumptions, the following nondimensional equations of motion are obtained:

$$\Lambda'' - \Lambda\phi'^2 - 2\Lambda\phi' - 3\Lambda\cos^2\phi = -\hat{T} \quad \text{(1a)}$$

$$\phi'' + 2(\Lambda'/\Lambda)\phi' + 3\sin\phi\cos\phi + 2(\Lambda'/\Lambda) = 0 \quad \text{(1b)}$$

where $(\ )' = d(\ )/d\tau$, $\tau = \Omega t$, t is time, and Ω is the angular velocity of the satellite in its orbit; $\Lambda = \ell / |\ell(t=0) - \ell_m|$, ℓ_m is the desired length for the deployment or retrieval; and $\hat{T} = T/[m\Omega^2 |\ell(t=0) - \ell_m|]$.

It may be noted that Eqs. (1) are nonlinear and time vaying as they contain the time-varying parameters ℓ and ℓ'.

Mission

The present problem of deployment and retrieval is described by the following mission: The mission is to change the state from the initial one to the final state $\ell = \ell_m$, $\ell' = \phi = \phi' = 0$.

Mission Function Control

The purpose of the controller is to stabilize the equilibrium state determined by the mission. Call this equilibrium state the mission state. The mission function control is designed to stabilize the mission state by generating a Lyapunov function with the mission state as the equilibrium state.

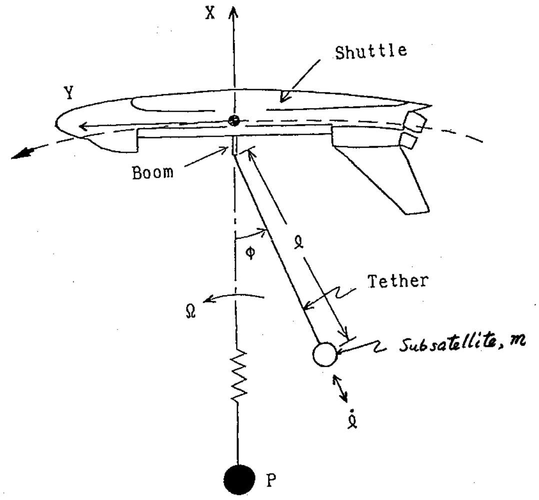


Fig. 1 Schematic representation of tethered satellite system.

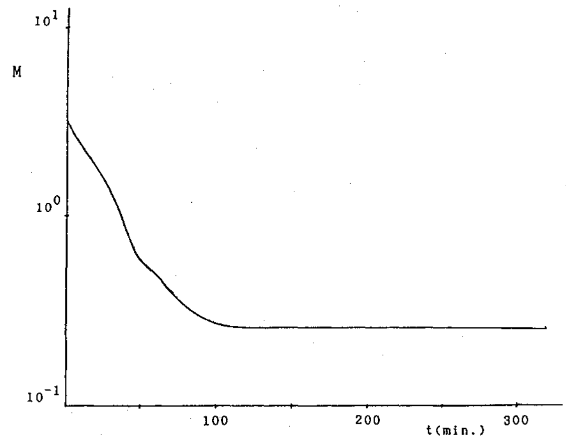


Fig. 2 Plots on a log scale of mission function against time.

The mechanical system S treated here is assumed to consist of two subsystems, namely, S_1 with control force and S_2 without control force, represented by the following equations of motion:

$$S_1: \ddot{x} + f(x, \dot{x}, y, \dot{y}) = u \quad \text{(2a)}$$

$$S_2: \ddot{y} + g_1(x,\dot{x},y,\dot{y})\dot{x} + g_2(x,\dot{x},y,\dot{y})\dot{y} + g_3(x,\dot{x},y,\dot{y}) = 0 \quad \text{(2b)}$$

where x and y are the state vectors, u denotes the control force, and $(\dot{\ })$ denotes the derivative with respect to time.

Let the mission function be specified as

$$M = cH + \tfrac{1}{2} a_1 \dot{x}^T \dot{x} + a_2 F(x) + \tfrac{1}{2} b_1 \dot{y}^T \dot{y} + b_2 G(x, y) \quad \text{(3)}$$

where H is the Hamiltonian of the system modified to be positive-definite and zero only at the mission state; $F(x)$ and $G(x,y)$ are positive-definite functions and are zero at the mission state; c, a_i, b_i, (≥ 0) are weighting coefficients, and $(\ )^T$ denotes the transpose.

The mission function M is positive-definite and is zero only at the mission state, i.e.,

$$\dot{x} = \dot{y} = 0, \qquad x = x_m, \qquad y = y_m \quad \text{(4)}$$

where the subscript m denotes the value at the mission state.

Differentiation of the mission function with respect to time gives

$$\frac{dM}{dt} = -\dot{x}^T\tilde{u} - b_1\dot{y}^T g_2\dot{y} - \dot{y}^T\left(b_1 g_3 - b_2\frac{\partial G}{\partial y}\right) \tag{5}$$

where

$$\tilde{u} = -c\frac{\partial^2 L}{\partial \dot{x}^2}\, u - a_1(u - f) - a_2\frac{\partial F}{\partial x} + b_1 g_1\dot{y} - b_2\frac{\partial G}{\partial x} \tag{6}$$

and $\tilde{u}$ is the freely assignable part of the control force u, L is the Lagrangian, and $\partial^2 L/\partial \dot{x}^2 = (\partial^2 L/\partial \dot{x}_i^2, \cdots, \partial^2 L/\partial \dot{x}_i^2, \cdots)$.

The time derivative of the mission function is negative-definite when

$$g_2 \geqq 0, \qquad \dot{x}^T\tilde{u} > 0, \qquad b_1 g_3 - b_2\frac{\partial G}{\partial y} = 0 \tag{7}$$

The freely assignable part of the control force can be selected to satisfy these conditions, e.g., by setting $\tilde{u} = \dot{x}$. Therefore, if the conditions of Eq. (7) are satisfied, it is clear that the mission state is stable and the control u will accomplish the mission.

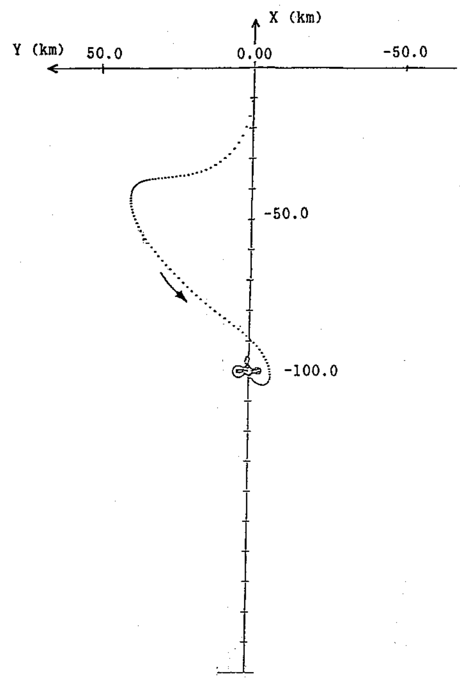


Fig. 3a Deployment of subsatellite.

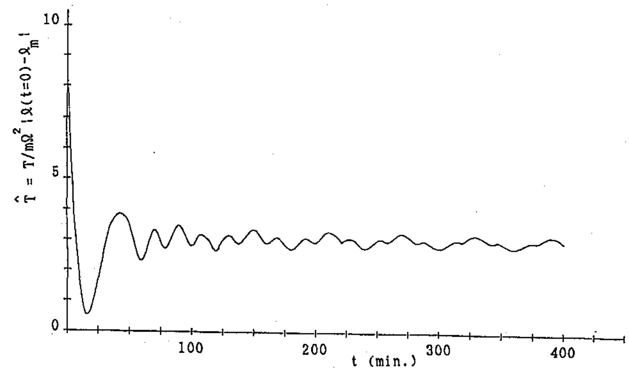


Fig. 3b Variation of nondimensional tension during deployment.

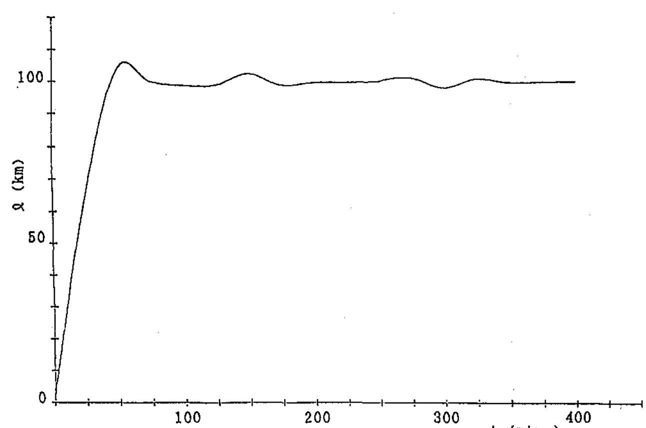


Fig. 3c Variation of length during deployment.

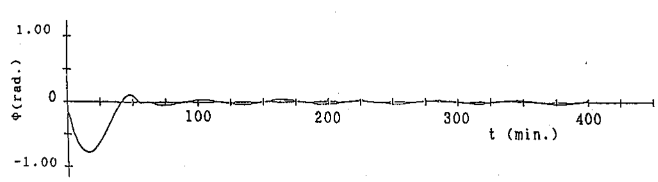


Fig. 3d Variation of rotation during deployment.

Deployment and Retrieval

The problem of synthesizing the control law for the given system and the given mission is reduced to find the appropriate or optimal (if possible) mission function and the freely assignable part of the control force.

For the present problem, the mission function is selected as follows:

$$M = \frac{1}{2}\left[a_1\Lambda'^2 + \left(e^{a_2(\Lambda-\Lambda_m)^2} - 1\right) + b_1\left(\frac{\phi'}{\sqrt{3}\,\sin(\pi/4)}\right)^2 + b_2\left(\frac{\sin\phi}{\sin(\pi/4)}\right)^2\right] \tag{8}$$

where the second term of M is introduced to emphasize the process of the deployment and retrieval, and the rotational angle ϕ is normalized with respect to a rotation of 45 deg and ϕ' with multiplied by $\sqrt{3}$ of that of ϕ. The mission function consists only of the generalized energy functions and does not include the Hamiltonian for the current selection.

Using Eqs. (1), one obtains the derivative of the mission function with respect to the nondimensional time as

$$\frac{dM}{d\tau} = a_1\Lambda'(\Lambda\phi'^2 + 2\Lambda\phi' + 3\Lambda\cos^2\phi - \hat{T})$$
$$+ a_2(\Lambda - \Lambda_m)\Lambda' e^{a_2(\Lambda-\Lambda_m)^2} + \tfrac{2}{3}\, b_1\phi'(-2\Lambda'\phi'/\Lambda$$
$$-3\sin\phi\cos\phi - 2\Lambda'/\Lambda) + 2b_2\phi'\sin\phi\cos\phi \tag{9}$$

or, selecting $b_1 = b_2$

$$\frac{dM}{d\tau} = -\Lambda' a_1\tilde{T} \tag{10}$$

where

$$\hat{T} = \Lambda\phi'^2 + 2\Lambda\phi' + 3\Lambda\cos^2\phi + \frac{a_2}{a_1}(\Lambda - \Lambda_m)e^{a_2(\Lambda-\Lambda_m)^2}$$
$$-\frac{4}{3}\frac{b_1}{a_1}\phi'(\phi' + 1)/\Lambda + \tilde{T} \tag{11}$$

and $\tilde{T}$ is the freely assignable part of $\hat{T}$.

There are some choices of $\tilde{T}$ and its selection, for example, as

$$\tilde{T}=(k/a_1)\Lambda\Lambda' M, \qquad (k>0) \tag{12}$$

gives

$$dM/d\tau = -k\Lambda\Lambda'^2 M \tag{13a}$$

then

$$M(\tau)=M(0)\exp\left(-\int_0^\tau k\,\Lambda\Lambda'^2\,d\tau\right) \tag{13b}$$

A representative change of mission function with respect to time is shown in Fig. 2 (plots on the log scale) while the system is under the present control, and it is seen that the value of the mission function decreases as time increases and the control algorithm works quite well. The decrease of the mission function is rather slow after 100 min because the control law [Eq. (12)] is not changed throughout the process to present the fundamental aspects of the mission-function control.

Numerical Results

The Shuttle is assumed to follow a circular orbit with a radius of 6220 km and an orbital velocity of 7.065×10^{-2} rad/min.

The deployment problem is set with the initial conditions as follows:

$$\ell=1\text{ km},\qquad \dot{\ell}=12.66\text{ km/min},\qquad \phi=\dot{\phi}=0\text{ at }t=0 \tag{14}$$

and with the mission state as follows:

$$\ell_m=100\text{ km},\qquad \dot{\ell}_m=\phi_m=\dot{\phi}_m=0 \tag{15}$$

It may be noted that the initial deployment rate $\dot{\ell}$ is not restricted by a limit posed as in the case of deployment with $\dot{\ell}/\Omega\ell$ constant for the tether not to rotate around the Shuttle.[1]

Figure 3a presents the time history during deployment of the subsatellite position in the X–Y plane with $a_1=a_2=5.0$, $b_1=1.0$, $k=100.0$, respectively. Figures 3b, 3c, and 3d present the associated time histories of the nondimensional tension $\hat{T}$, length ℓ, and angle ϕ, respectively, of the tether. The deployment objective is accomplished after approximately 50 min. Note from Eqs. (1) that the effectiveness of the tension control to suppress the rotation ϕ decreases as the tether

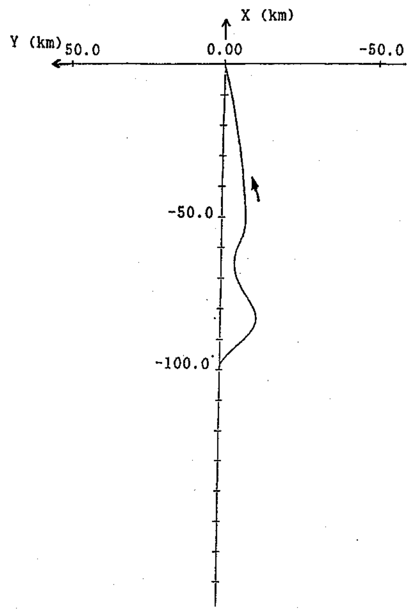


Fig. 4a Retrieval of subsatellite.

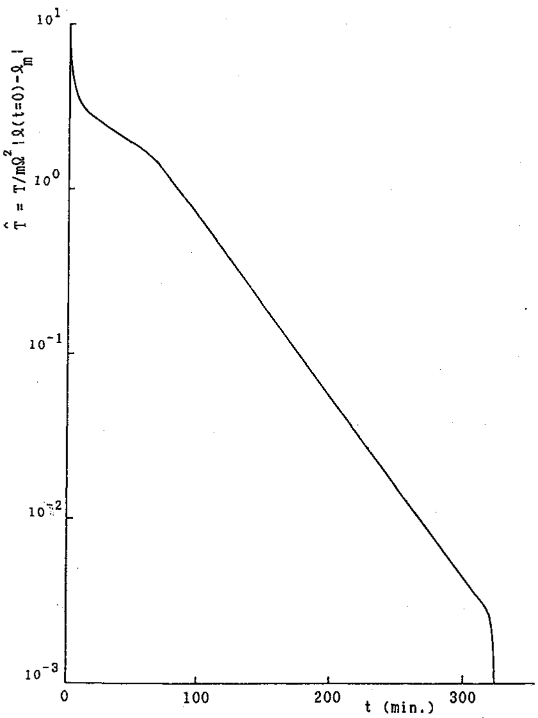


Fig. 4b Variation of nondimensional tension during retrieval (plots on a log scale).

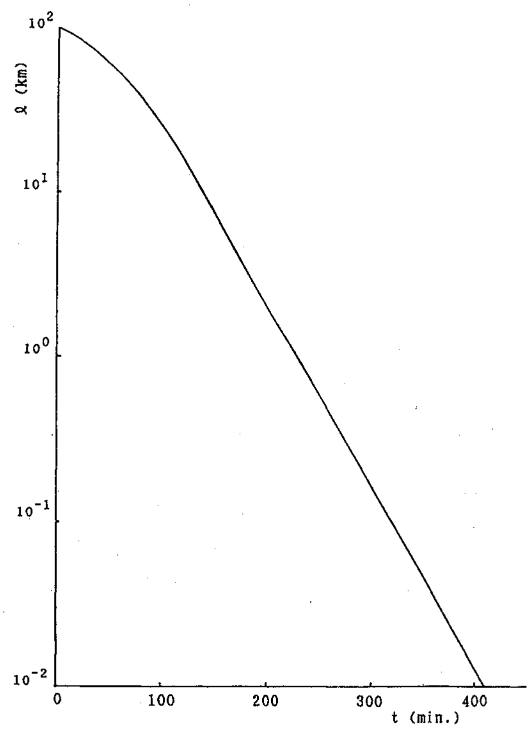


Fig. 4c Variation of length during retrieval (plots on a log scale).

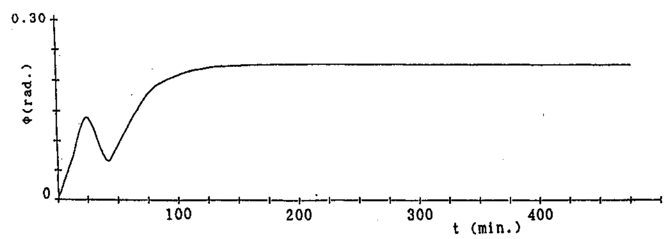


Fig. 4d Variation of rotation during retrieval.

approaches the mission state, since the tension acts on Eq. (1b) through nonlinear terms in Eqs. (1).

The retrieval case is shown in Fig. 4. The initial conditions and the mission state are selected as follows:

$$\ell = 100 \text{ km}, \qquad \dot{\ell} = \phi = \dot{\phi} = 0 \text{ at } t = 0 \tag{16}$$

$$\ell_m = 1 \text{ km}, \qquad \dot{\ell}_m = \phi_m = \dot{\phi}_m = 0 \tag{17}$$

The time locus of the subsatellite in the X-Y plane is shown in Fig. 4a. In this case, the values of the weighting coefficients are $a_1 = 0.5$, $a_2 = 2.0$, $b_1 = 5.0$, and $k = 100.0$. Figures 4b, 4c, and 4d (plots on the log scales) show the change with respect to time of the associated nondimensional tension, length of tether, and angle, respectively. The tether becomes slack when the value of the tension is negative, which sometimes produces failure of the retrieval control algorithm.[5,6] However, the results show the present algorithm to be effective over the first 325 min, maintaining positive tension; the subsatellite is within the capture range of 80 m at that time. It may also be noted that the current deployment and retrieval processes need less time than those previously presented because these processes will almost be accomplished within at least four orbits.[4]

Conclusion

A new control algorithm is presented called the mission function control. The control algorithm makes the desired mission state into an equilibrium state and then stabilizes the mission state. A positive-definite function is selected as the mission function, which is zero at the mission state. The control is then performed to reduce the value of the mission function as time increases, i.e., the time derivative of the mission function is negative-definite.

The algorithm is applied to the problem of deployment and retrieval of a subsatellite. The results of numerical simulation show excellent controlled behavior during deployment and retrieval.

It may be noted that the present paper employed a rather simplified dynamical model, only in-plane motion is treated, and neither flexibility nor air drag is included. The controllability of a more realistic model must still be explored.

Acknowledgment

The authors are indebted to Professor R. W. Longman of Columbia University for having read and criticized the manuscript.

References

[1]Kulla, P., "Dynamics of Tethered Satellites," *Dynamics and Control of Non-Rigid Spacecraft,* ESA SP 117, European Space Agency, May 1976, pp. 349–354.

[2]Rupp, C. C. and Laue, J. H., "Shuttle/Tethered Satellite System," *Journal of Astronautical Sciences,* Vol. 26, Jan.–March 1978, pp. 1–17.

[3]Misra, A. K. and Modi, V. J., "Dynamics and Control of Tether Connected Two-Body Systems—A Brief Review," *Space 2000,* edited by L. G. Napolitano, AIAA, New York, 1983, pp. 473–514.

[4]Banerjee, A. K. and Kane, T. R., "Tethered Satellite Retrieval with Thruster Augmented Control," *Journal of Guidance, Control, and Dynamics,* Vol. 7, Jan.–Feb. 1984, pp. 45–50.

[5]Xu, D. M., Misra, A. K., and Modi, V. J., "Thruster-Augmented Active Control of a Tethered Subsatellite System During its Retrieval," *Journal of Guidance, Control, and Dynamics,* Vol. 9, Nov.–Dec. 1986, pp. 663–672.